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RESEARCH ARTICLE

ABOUT ONE BINARY PROBLEM IN A CLASS OF ALGEBRAIC EQUATIONS AND HER COMMUNICATION WITH THE GREAT HYPOTHESIS OF FERMAT

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A R T I C L E I N F O

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A B S T R A C T

In this article the author introduces the notion of the binary mathematical statement A_n from natural parameter n and refined axiomatic Peano natural numbers by adding the axiom of descent which is algebraic interpretation of the so-called method of descent Fermat. The known class of the Diophantine equations Fermat is reduced to some class of the algebraic equations from natural parameter n, $n \geq 3$ (degree of polynomial). It is proved that concerning binary statement B_n: "whether has the equation for a preset value n some decision x_n" the corresponding classes of the algebraic and Diophantine equations are equivalent. We show that the constructed class of the algebraic equations has rational decision only for $n = 4$. For $n = 3, 4$ the constructed class of the algebraic equations has decisions in radicals, and for $n \geq 5$ this classes of the equations isn't solvable at all. Thus also it is prove, that the Great Hypothesis of Fermat is correct with the small precision: the class Diophantine equations of Fermat have not the decision non-only in integers but and in the rational field.

INTRODUCTION

In 1995, in the Annals of Mathematics, it published an article [1] that caused a lot of noise in the Western press and some publications, for example in the form of monographs [2] and on the internet [3].

In [4] it was stated that in [1] used in the proof only Euclidean geometry (parabolic classification F.Klein [5]), which can not be considered a complete proof, as Euclidean geometry is a very special case of nonEuclidean geometry, when the angle of parallelism rate $\frac{\pi}{2}$, the while in other cases it is an acute [5,31].

To confirm our doubts as to the completeness proof Fermat's theorem in [1], we have considered in [6], especially of natural numbers in different geometries and found that the properties of the natural numbers depend on the geometry of the system of axioms (see [6], Theorem 1,2,3).

Another reason why we can not agree this proof in [1] Fermat's Last Theorem is that the author [1] uses to prove the same method that was used before him by other authors (Cauchy, Lame, Kummer, etc.) who subsequently were found insurmountable gaps in the evidence.

Elliptic curves, which are used in [1] the author and attracts Euclidean geometry can not be an excuse.

Decision of class of Diophantine equations of Fermat in the field of complex numbers for $3 \leq n < 5$ and them insolvability for $n \geq 5$

In [7-8] we introduced concept of the binary mathematical statement and refined the axiomatic of natural numbers Peano [9], having added to system of axioms an axiom of descent which is algebraic interpretation of a method of descent of Fermat [10]. In [7] we also announced the results received in the real work on Fermat's problem.

Definition 1 [7]. An mathematical statement A_n, depending on natural parameter n we will call binary if for any value $n = \alpha$ the statement A_α has one or the other values: truth or lie.

Axiom of descent [7]: let A_n will be the binary statement from natural parameter n such that:

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1. there is an algorithm which for any value n gives the answer to the question "statement A_n truth or lie?";
2. for values of parameter $n_1 < n_2 < ... < n_k$ the statements $A_{n_1}, A_{n_2}, ..., A_{n_k}$

are true, and for any $n_{k+1} > n_k$ the statement $A_{n_{k+1}}$ is false.

Then the statement A_n is true for infinitely many values n.

With use of this axiom we in [7-8] have proved a lot of the numbers of problems opened in theory of numbers some of which age reaches more, than 2500.

We will consider a class of the algebraic equations

$$(x-2)^n + (x-1)^n = x^n \quad (1)$$

and class of the Diophantine equations

$$u^n + v^n = w^n \quad (2)$$

from natural parameter $n \geq 3$.

Theorem 1. The equations (1), (2) concerning the statement A_n: "whether the decision for some n has the equation?" are equivalent.

Proof. Really, if the equation (1) for some $n = k$ has the solution x_k, then the equation (2) for this $n = k$ also has the decision. To obtain the corresponding solution to equation (2) it is enough to put $u_k = x_k - 2, v_k = x_k - 1$, $w_k = x_k$.

According to the logical law of contraposition, if the equation (2) is not solvable for $n = k$, then the equation (1) for $n = k$ is also not solvable. If also the equation (2) for some $n = k$ has the decision u_k, v_k, w_k, then the equation (1) for this $n = k$ has the solution $x_k = w_k$. Therefore, equation (1), (2) are equivalent with respect to the specified affirmation A_n. As classes of the equations (1), (2) concerning the statement A_n are equivalent, all statement proved for the equation (1) will be fair for the equation (2).

Let B_n will be the assertion: the equation (1) for n has no rational decisions. Obviously, B_n is a binary statement. Indeed, for any particular values of $n = k$ the equation

$$(x-2)^k + (x-1)^k = x^k \quad (3)$$

or has a rational solution or not. In the first case B_k is false and the second true. You can check [4,6] that for $k = 1, 2$ the statement B_k is false, and for $k = 3$ is true. Obviously, this verification process can continue for $k = 4, 5, 6, ...$ Thus, we are in the conditions of the axiom of descent, i.e. to determine that the statement B_k is true or false there is an algorithm which consists in verifying whether has among the divisors of the free term of equation (3) the solution of this equation. If so, B_k is false, if not, then B_k is true. For $k = 4$ the equation (3) will take a form

$$x^4 - 12x^3 + 30x^2 - 36x + 17 = 0 \quad (4)$$

Obviously the equation (4) have the rational decision $x = 1$. Therefore the correspondent decision of the equation (2) will be: $u = 1, v = 0, w = 1$. The resulting solution of Fermat's equation does not satisfy the terms of Fermat's theorem as $v = 0$.

Theorem 2.[11] The algebraic equation (1) for $n = 3, 4$ have the solution in field of complex numbers.

Hence by theorem 1:

Corollary 1. The Diophantine equation (2) for $n = 3, 4$ have the decisions in field of complex numbers.

Theorem 3. Algebraic equation (1) for all $n \geq 5$ algorithmically is insolvable.

Proof. For $n = 5$ the equation (1) will take a form

$$x^5 - 15x^4 + 50x^3 - 90x^2 + 85x - 33 = 0$$

The received equation by means of replacement of unknown $x = y + 1$ will be transformed to a look

$$y^5 - 10y^4 - 20y^2 - 2 = 0 \quad (5)$$

According to Eisenstein [11,353] criterion the left member of the equation (5) is the prim polynomial over field of rational numbers. From there is the equation (5), and the equation (1) for $n = 5$ is insolvable. In the thirties of the 19th centuries Galois was succeeded to find [11,353] conditions under which this algebraic equation is solvable in radicals. We will assume now the equation (1) for $n = 6, 7, ..., t$ is insoluble, and for $n = t + 1$ is solvable. Thus, we are in descent axiom conditions. Therefore, according to a axiom of descent the equation (1) for $n = t$ is also solvable, and it contradict our inductive assumption. Therefore the equation (1) for $n \geq 5$ is insoluble. From here according to the theorem 1 follows.

Corollary 2. The Diophantine equation (2) for $n \geq 5$ is algorithmically unsolvable.

Obviously justice of the Great hypothesis of Fermat as a result follows from the results received above.

CONCLUSIONS

1. We have proved more, than is announced in the Great Hypothesis of Fermat: Fermat's equations have no decision not only in integers, but also in the field of rational numbers.
2. Reduction of the equations of Fermat to the elliptic equation of Frey isn't correct as Fermat's equations have no decisions in integers
3. We have constructed algorithm of the proof of the statement (watch point 1.) for any n, including compound n whereas others built the proofs for some concrete n. In this regard we will specify a case of n=4 for which we have found the rational solution of

the equation (1) which isn't meeting conditions of the statement of Fermat.

4. We have found solutions of the equations of Fermat in radicals for n=3,4 and have proved their insolvability for $n \geq 5$.
5. Now there is an opportunity, taking into account the remarks specified in introduction, to compare informational content of the results received in the real work and in work [1] especially, meaning their volume in numbers of the used pages and their readability [3].
6. At last, we will note that we investigated the subject touched by Pierre Fermat not for the sake of what satisfaction that vain ambitions, and for the sake of knowledge of the mathematical truth opened in a little cut down form by Pierre Fermat that, is certainly clear as this statement has been formulated in the 17th century.
7. We will note still that with an appearance of article [1] with Volfskel's will, unfortunately, have disposed unfairly.
8. In the history of mathematics strange cases are known:

   a. so, for example, the 21 st problem of D.Gilbert "has been positively solved" at the beginning of the 20th century by the mathematician E.Plemel, but in 70 years the Russian mathematician A.A.Bolibrukh has noticed an error in E.Plemel's proof and has solved D.Gilbert's problem "on the contrary"- it has turned out that this problem has the negative decision [12,129];
   b. it seemed to mathematics that the fifth postulate of Euclid is a theorem which Euclid couldn't prove. It was the problem with which solution subsequently more 2000 unsuccessfully struggled hundreds of geometry [5,20]. In the report of 1826 which has made an era in development of geometry Nikolay Ivanovich Lobachevsky has given already final but absolutely unexpected solution of the problem of parallels, having created new non-Euclidean geometry and strangely enough this opening of Lobachevsky has found the recognition in 30 years after his death.

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