INTERNATIONAL JOURNAL OF
CURRENT MULTIDISCIPLINARY STUDIES

Available Online at http://www.journalijcms.com
Vol. 2, Issue, 2, pp. 130-133, February, 2016

IJCMS

RESEARCH ARTICLE

AN INTERESTING DIOPHANTINE PROBLEM ON TRIPLES-II

Gopalan M.A., Vidhyalakshmi S., Shanthi J and Rukmani A

Department of Mathematics, SIGC, Trichy

ARTICLE INFO

Article History:
Received 15th, December, 2015
Received in revised form
28th, December, 2015
Accepted 16th, January, 2016
Published online
28th, February, 2016

Keywords:

System of linear equations, Integer solutions, Cubic with four unknowns. 2010 MSC classification:11D04, 11D25, 11D99.

ABSTRACT

This paper concerns with an interesting Diophantine problem and aims at determining explicitly three distinct non-zero integers a,b,c such that $a+2b=\alpha^2, a+2c=\beta^2, b+c=\gamma^2$ and sum of the three integers is a perfect square. Different methods have been considered to obtain the three required integers a,b,c . This shows that there are many triples in integers, each satisfying the considered kind of pattern among its members.

INTRODUCTION

Number theory, called the Queen of Mathematics, is a broad and diverse part of Mathematics that developed from the study of the integers. The foundations for Number theory as a discipline were laid by the Greek mathematician Pythagoras and his disciples (known as Pythagoreans). One of the oldest branches of mathematics itself is the Diophantine equations since its origins can be found in texts of the ancient Babylonians, Chinese, Egyptians, Greeks and so on[1-6]. Diophantine problems were first introduced by Diophantus of Alexandria who studied this topic in the third century AD and he was one of the first Mathematicians to introduce symbolism to Algebra. The Greek mathematician Diophantus is renowned for his work on solving equation with rational solutions. Number Theorists relish tackling Diophantine equations with integer solutions. There is still much interest today in Diophantine problems and has been a topic of keen interest to many mathematicians worldwide because of its historical importance.

They offer good applications in the fields of patterns classification, graph theory, elliptic curves, modular forms, galois representation, engineering, coding, cryptography, music, analysis of non-linear resonances in fluid mechanics and so on. The beauty of many Diophantine equations lies in the fact that they are easy to understand, yet very difficult to solve. Fermat's Last Theorem is an example to illustrate this point.

*✉ ***Corresponding author:*** **Gopalan M.A**
Department of Mathematics, SIGC, Trichy

If we can describe some phenomena carefully enough as a pattern, then mathematics may be able to give information about that kind of pattern. Perhaps only a mathematician would think of the empty set as a pattern- the "null patterns". In fact, Diophantine problems dominated most of the celebrated unsolved mathematical problems. Certain Diophantine problems come from physical problems or from immediate Mathematical generalizations and others come from geometry in a variety of ways. Certain Diophantine problems are neither trivial nor difficult to analyze [7-8]. In this context, one may also refer [9-14]. The above results motivated us to search for integer solutions to various other choices of Diophantine problems.

This paper aims at constructing an interesting Diophantine problem and aims at determining explicitly three distinct non-zero integers a,b,c such that $a+2b=\alpha^2, a+2c=\beta^2, b+c=\gamma^2$ and sum of the three integers is a perfect square.

METHOD OF ANALYSIS

The Diophantine problem under consideration is to solve the system of equations

$$a+2b=\alpha^2 \quad (1)$$

$$a+2c=\beta^2 \quad (2)$$

$$b+c=\gamma^2 \quad (3)$$

$$a+b+c=\delta^2 \quad (4)$$

where a,b,c represents the three non -zero integers.

Solving the above equations (1) and (2), we get the values of a, b, and c to be

$$a=\frac{1}{2}(\alpha^2+\beta^2-2\gamma^2)$$

$$b=\frac{1}{4}(2\gamma^2+\alpha^2-\beta^2)$$

$$c=\frac{1}{4}(2\gamma^2-\alpha^2+\beta^2)$$

For a, b, c to be an integers, substituting $\alpha=2p,\beta=2q,\gamma=2r$ in the above equations, we get

$$a=2p^2+2q^2-4r^2 \quad (5)$$

$$b=2r^2+p^2-q^2 \quad (6)$$

$$c=2r^2-p^2+q^2 \quad (7)$$

Substituting the above values in (4), we get

$$2(p^2+q^2)=\delta^2 \quad (*)$$

Method –I

First introducing the transformations

$$p=u+v, q=u-v \quad (8)$$

From (*), we get

$$\delta^2=(2u)^2+(2v)^2 \quad (9)$$

which is in the form of Pythagorean equation and it is satisfied by

$$u=st, v=\frac{1}{2}(s^2-t^2), \delta=s^2+t^2 \quad (10)$$

Case I

Consider s and t are even so that $s=2S, t=2T$

$\therefore$ The values of u, v and δ are

$$u=4ST, v=2(S^2-T^2), \delta=4(S^2+T^2)$$

Substituting the values of u ,v in (8), we get

$$p=2S^2-2T^2+4ST \quad (11)$$

$$q=2T^2-2S^2+4ST \quad (12)$$

Substituting the values of p and q in (5), (6) and (7), we get

$$a=16(S^2+T^2)^2-4r^2$$

$$b=2r^2+16ST(2S^2-2T^2)$$

$$c=2r^2-16ST(2S^2-2T^2)$$

A few numerical examples are presented in the table below.

Table-I

S	T	r	A	b	c	*a+2b*	*a+2c*	*b+c*	*a+b+c*
1	2	1	396	-190	194	16	784	4	400
3	4	2	9984	-2640	2696	4624	15376	16	10000
5	6	3	59500	-10542	10578	38416	80656	36	59536
7	8	4	204240	-26848	26912	150544	258064	64	204304

Case-II

Consider s and t are odd so that $s=2S+1, t=2T+1$

We get the values of u, v and δ to be,

$$u=4ST+2S+2T+1$$

$$v=2(S^2-T^2+S-T)$$

$$\delta=(2S+1)^2+(2T+1)^2$$

The values of u ,v in equation (8), we get

$$p=2S^2-2T^2+4ST+4S+1 \quad (13)$$

$$q=2T^2-2S^2+4ST+4T+1 \quad (14)$$

Substituting the values of p and q in (5), (6) and (7), we get

$$a=(4S^2+4T^2+2)^2+(4S+4T)^2+16S(1+2T^2)+16T(1+2S^2)+32S^3+32T^3-4r^2$$

$$b=2r^2+24S^2+32S^2T-32ST^2+8S-8T+32S^3T+16S^3-16T^3-32ST^3-24T^2-16ST^2+16S^2T$$

$$c=2r^2-24S^2-32S^2T+32ST^2-8S+8T-32S^3T-16S^3+16T^3+32ST^3+24T^2+16ST^2-16S^2T$$

A few numerical examples are presented in the table below.

Table-II

S	T	r	A	b	c	a+2b	a+2c	b+c	a+b+c
1	2	1	1152	-478	482	196	2116	4	1156
3	4	2	16884	-4024	4040	8836	24964	16	16900
5	6	3	84064	-13710	13746	56644	111556	36	84100
7	8	4	264132	-32608	32672	198916	329476	64	264196

Method-II

Assume that $\delta=2w$

From (*), we get

$$p^2 + q^2 = 2w^2 \qquad (15)$$

Case-I

Now '2' can be written as

$$2 = (1+i)(1-i) \quad \text{and} \quad w = (A+iB)(A-iB) \qquad (16)$$

Then,

$$p + iq = (1+i)(A+iB)^2$$

$$p + iq = A^2 - B^2 + 2iAB + iA^2 - iB^2 - 2AB \qquad (17)$$

Equating real and imaginary parts of the above equation, we get

$$p = A^2 - B^2 - 2AB \qquad (18)$$

$$q = A^2 - B^2 + 2AB \qquad (19)$$

Substituting p and q values in (5),(6) and (7), we get

$$a = (2A^2 + 2B^2)^2 - 4r^2$$
$$b = 2r^2 + 8AB[B^2 - A^2]$$
$$c = 2r^2 + 8AB[A^2 - B^2]$$

A few numerical examples are presented in table below

Table-III

A	B	r	a	b	c	a+2b	a+2c	a+b	a+b+c
1	2	1	96	50	-46	196	4	4	100
3	4	2	2484	680	-664	3844	1156	16	2500
5	6	3	14848	2658	-2622	20164	9604	36	14884
7	8	4	51012	6752	-6688	64516	37636	64	51076

Case II

Again '2' can be written in the form,

$$2 = \frac{(7+i)(7-i)}{25}$$

Then,

$$p + iq = \frac{(7+i)}{5}(A+iB)^2$$

$$5p + 5iq = 7A^2 - 7B^2 + 14iAB + iA^2 - iB^2 - 2AB \qquad (20)$$

Equating real and imaginary parts of the above equation, we get

$$p = \frac{1}{5}(7A^2 - 7B^2 - 2AB)$$

$$q = \frac{1}{5}(A^2 - B^2 + 14AB)$$

Substitute A=5A and B=5B in the above equations we get the values of p and q to be

$$p = 7A^2 - 7B^2 - 2AB \qquad (21)$$

$$q = A^2 - B^2 + 14AB \qquad (22)$$

Substituting p and q values in (5),(6) and (7), we get

$$a = (10A^2 + 10B^2)^2 - 4r^2$$
$$b = 2r^2 + 48A^4 + 48B^4 - 288A^2 + 56AB^3 - 56A^3B$$
$$c = 2r^2 - 48A^4 - 48B^4 + 288A^2 - 56AB^3 + 56A^3B$$

A few numerical examples are presented in table below

Table-IV

A	B	r	a	b	c	a+2b	a+2c	a+b	a+b+c
1	2	1	2496	2	2	2500	2500	4	2500
3	4	2	62484	-20584	20600	21316	103684	16	62500
5	6	3	372064	-148494	148530	20164	669124	36	14884
7	8	4	1276836	-544240	544304	188356	2365444	64	51076

CONCLUSION

In this paper, we have presented an interesting diophantine problem of determining explicitly three non-zero distinct integers a, b, c such that $a + 2b = \alpha^2$, $a + 2c = \beta^2$, $b + c = \gamma^2$ and $a + b + c = \delta^2$ .The reader will now be desirous to become acquainted with the classes of indeterminate problems which Diophantus treats and his method of solution Diophantus gives only the most special solutions of all the questions which he treats and he is generally content with indicating numbers which furnish one single solution. But it must not be supposed that his method was restricted to these very special solutions. In conclusion, one may investigate several further and new explicit diophantine problems.

References

1. André Weil, (1987)" Number Theory: An Approach through History, From Hammurapito to Legendre", Birkahsuser, Boston.
2. BibhotibhusanBatta and Avadhesh Narayanan Singh, (1938) "History of Hindu Mathematics", Asia Publishing House.
3. Boyer.C.B., (1968) "A History of mathematics", John Wiley& sons Inc., NewYork.
4. Dickson L. E., (1952)" History of Theory of Numbers", Vol. 11, Chelsea Publishing Company, New York.
5. Davenport.H (1999)," The higher Arithmetic: An Introduction to the Theory of Numbers"(7th ed.) Cambridge University Press.

6. John Stilwell, (2004) "Mathematics and its History", Springer Verlag, New York.
7. James Matteson, M.D, (1888)"A Collection of Diophantine problems with solutions", Washington, ArtemasMartin.
8. Tituandreescu, DorinAndrica, (2002) "An Introduction to Diophantine equations" GIL Publication House.
9. M.A. Gopalan, S. Vidhyalakshmi, N. Thiruniraiselvi, (2015) "On Two Interesting Diophantine Problems", Impact Journal of Science and Technology, Vol-9, No-3,51-55
10. M.A. Gopalan, S. Vidhyalakshmi, N. Thiruniraiselvi, (2015) "On Interesting Integer Pairs",IJMRME, Vol-1, ISS-1,320-323.
11. M.A. Gopalan, S. Vidhyalakshmi, J. Shanthi, (2015) "On Interesting Diophantine Problem", IJMRME, Vol-1, ISS-1, 168-170.
12. M.A. Gopalan, S.Vidhyalakshmi, J. Shanthi, (2015) " An Interesting Diophantine Problem", Scholar Bulletin, Vol-1, ISS-7, 166-168.
13. K.Meena, S.Vidhyalakshmi, J.Shanthi, K.Agalya " An Interesting Diophantine Problem", IJREAS, Vol-5, ISS-12, (Dec,2015),93-98
14. S.Vidhyalakshmi, E.Premalatha, R.Presenna, V.Krithika, (2016) "Construction of a special integer triplet-I, IJPMS, Vol-16,26-29.

How to cite this article:
Gopalan M.A *et al*.,: An Interesting Diophantine Problem On Triples-II, International Journal of Current Multidisciplinary Studies 2016; 2(2): 130-133.